



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

But this formula does not enable us to obtain least values of  $p$ ,  $q$ , and  $m$ , as  $n$  varies.

56. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

If  $\phi(R)$  is the number of integers which are less than  $R$  and prime to it, and if  $y$  is prime to  $R$ , show that  $y^{\phi(R)} - 1 \equiv 0 \pmod{R}$ .

Solution by the PROPOSER, and J. O. MAHONEY, B. E., M. Sc., Lynnville, Tenn.

Let  $1, m, n, p, \dots (R-1)$  denote the  $\phi(R)$  numbers less than  $R$  and prime to it; now  $y$  can be any one of those numbers.

$\therefore y, my, ny, py, \dots (R-1)y$  are all prime to  $R$  and all different.

There are  $\phi(R)$  of such products and since when these products are divided by  $R$  the remainders are all prime to  $R$  and all different, the  $\phi(R)$  remainders must be  $1, m, n, p, \dots (R-1)$  though not necessarily in this order.

$\therefore y.my.ny.py \dots (R-1)y$  must differ from  $1.m.n.p \dots (R-1)$  by a multiple of  $R$ .

$\therefore \{y^{\phi(R)} - 1\} mnp \dots (R-1) = \text{a multiple of } R$ .

But  $mnp \dots (R-1)$  is prime to  $R$ .

$\therefore y^{\phi(R)} - 1 \equiv 0 \pmod{R}$ .

57. Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

Each of *five* of the digits may be the terminal figure of a perfect integral square. Each of *eighteen* combinations of two digits may be the *two* terminal figures of an integral square. Each of *one hundred and nineteen* combinations of three digits may be the *three* terminal figures of an integral square. *Under these conditions*, what is the greatest number of arrangements of the nine digits, all taken together, whose three terminal figures shall be those of a square number?

No solution of this difficult problem has been received. Can any of our readers furnish the desired solution? EDITOR.

#### MISCELLANEOUS.

58. Proposed by J. A. CALDERHEAD, M. Sc., Professor of Mathematics in Curry University, Pittsburg, Pa.

(a) What is the highest north latitude in which the Sun will shine in at the north window of a building at least once in a year?

(b) How many days will it shine in at the north window of a building in latitude  $41^\circ$  N.?

Note by SAMUEL HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

Whenever the Sun, or any part of it, is north of the prime vertical, it must then shine on the north side of buildings. From the time of vernal equinox, to the autumnal equinox, the Sun will be north of the prime vertical during some part of every day, and will shine on the north side of buildings some part of *every* day for about half a year, and in *all* latitudes north of the equator. Hence the answer for (a) is  $90^\circ$  N. latitude, and for (b) 186 days, but if the Sun's upper

limb, and refraction, be considered the days will be 187 or 188. The answer for  $41^\circ$  N. is *good for any* other latitude north, while the problem seems to imply that an answer for  $41^\circ$  is different for other latitudes.

54. Proposed by S. HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

On latitude  $40^\circ$  N.  $=\lambda$ , when the Moon's declination is  $5^\circ 23'$  N.  $=\delta$ , and the Sun's  $9^\circ 52'$  S.  $=-\delta$ , how long after sunset will the two horns or cusps of the Moon's crescent (recently new) set at the same moment, the crescent with its back *down* having touched the horizon first? Semi-diameters, refraction, and parallax not considered.

#### I. Solution by the PROPOSER.

Let  $B$  be the celestial north pole,  $A$  the zenith,  $AB$  an arc of the meridian equal the co-latitude  $=c=50^\circ$ ,  $HO$  a portion of the horizon,  $SS'$  and  $MM''$  portions of the diurnal arcs of the Sun and Moon, the Sun setting at  $S$ , and the Moon at  $M'$ ;  $BS$  = the polar distance of the Sun  $=BS'$ , and  $BM'$  the polar distance of the Moon, and  $AM'$  the zenith distance of the Moon  $=90^\circ$ .

Produce the vertical circle  $AM'$  to  $S'$ ,  $S'$  being the place of the Sun when the Moon sets at  $M'$ . The line joining the Moon's cusps must be at right angles to the line  $M'S'$  joining the centers of the Sun and Moon, and as the horizon is at right angles to  $AM'S'$ , the line of the cusps must lie on the horizon and set when the Moon's center sets. Put  $\angle ABS = \phi$  = Sun's hour angle when it sets, and  $\angle ABS' = \theta$  = Sun's hour angle when the Moon sets, and  $\angle ABM' = \psi$  = Moon's hour angle when it sets.

Then we have  $\cos \phi = \tan \delta' \tan \lambda$ .  $\therefore \phi = 81^\circ 36' 29''$ , and  $\cos \psi = -\tan \delta \tan \lambda$ .  $\therefore \psi = 94^\circ 32' 7''$ . Take an auxiliary arc  $\chi'$ , and  $\tan \chi' = \cos \psi \cot \delta$ .  $\therefore \chi' = 40^\circ 0' 1''$ , then  $\cot A = \sin(c - \chi') \cot \chi' \operatorname{cosec} \chi'$ .  $\therefore A = 82^\circ 57' 55''$ . Take an auxiliary angle  $\gamma'$ , and  $\cot \gamma' = \tan A \sin \lambda$ .  $\therefore \gamma' = 10^\circ 52' 2''$ . Then  $\cos \gamma' \cot \lambda \tan -\delta' = -\cos \gamma$ .  $\therefore \gamma = 101^\circ 44' 43''$ , and  $\angle ABS' = \gamma' + \gamma = \theta = 112^\circ 36' 45''$ , and  $\theta - \phi = 31^\circ 0' 16'' = 2$  hours, 4 minutes, 1 second.

NOTE. The synchronous setting or rising of the cusps of a crescent Moon, is a phenomenon which must occur frequently in the tropics, and rarely or not at all beyond latitude  $45^\circ$ . On the 4th of July, 1897, such a moonset was very nearly accomplished, and another, almost perfect, will occur February 22, 1898, the declinations being then as given in the problem. Few persons in the northern states have ever seen the Moon set with both horns vertical.

II. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Let  $O$  be the observer,  $Z$  his zenith,  $HMK$  Moon's path,  $GCSL$  Sun's path,  $TEFR$  celestial equator,  $AMCB$  the horizon. Let  $M$  be the position of the Moon when setting. Then, in order that the horns may set at the same time,  $S$ ,  $M$ , where  $S$  is the Sun, must be on the same meridian,  $ZMSN$ .

$AP = \lambda = 40^\circ$ .  $ME = \delta = 5^\circ 23'$  N.  $SF = \delta_1 = 9^\circ 52'$  S. In the triangle

